Assignment MST-018

for

M.Sc. (Applied Statistics) (MSCAST)

Valid from January 2025 to December 2025

SCHOOL OF SCIENCES
Indira Gandhi National Open University
New Delhi - 110068

Dear Learner,

Welcome to the M.Sc. (Applied Statistics) Programme.

As per the university guidelines, you need to complete the assignment for each theory course. Note that there are no assignments for lab courses in the MSCAST programme, namely, MSTL-011, MSTL-012, MSTL-013, MSTL-014, and MSTL-015. You should remember that writing answers to an assignment's questions will improve your writing skills and prepare you for the term-end examination.

It is compulsory to submit the assignments within the stipulated time to be eligible to appear in the term-end examination. You will not be allowed to appear for the term-end examination for a course if you do not submit the assignment for that course by the due date. As per the University guidelines, if you appear in the term-end examination of a course without submitting its assignment, the result of the term-end examination is liable to be cancelled/ withheld.

The assignments constitute the continuous component of the evaluation process and have 30% weightage in the final grading.

Before you write the assignments, you are advised to first go through the self-learning material for that course and then prepare the assignments carefully by following the instructions pertaining to the assignments. Your responses should not be a verbatim reproduction of the textual materials provided for self-learning purposes, but it should be in your own words.

If you have any doubts or problems pertaining to the course material and assignments, contact the programme in charge or the academic counsellor at your study centre. If you still have problems related to this assignment, feel free to contact the course coordinator.

Wishing you all the best in successfully completing the programme.

(Dr. Taruna Kumari)
Course Coordinator, MST-018
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Instructions:

- Submit the assignments within the stipulated time. Otherwise, you will not be permitted to appear for the term-end examination.
- Solve the latest assignments uploaded for the current year/session.
- Read the instructions related to the assignments mentioned in the Programme Guide.
- Use only A-4 size paper to write your responses. It is mandatory to write all assignments neatly in your own handwriting. Typed or printed copies of the assignments will not be accepted. Note that you may use the printout only if a question specifically asks for the output of a program in MST-015 and MST-024.
- All questions given in the assignments are compulsory for each course.
- Express your response in your own words. You are advised to restrict your response based on the marks assigned to it. This will also help you to distribute your time in writing or completing your assignments on time.
- Securely fasten multiple pages together (you can staple or tie them) and number them carefully for each assignment separately.
- Do not forget to enclose the assignment question sheet of that course after the cover page of the assignment response (answer sheets). It is not compulsory to write each question separately before answering the question. Mention the question number for each answer.
- The solved assignment must be submitted at the Study Centre allotted to you before the due date set by the University. Please check the IGNOU website for updated information regarding the due date of assignment submission.
- You are advised to mention all information on the first page of the assignment response sheet, given on the next page.
- **Keep a copy of the assignment answer sheets with you before submission for future reference.**

ASSIGNMENT CODE: MST-018/TMA/2025

NAME: _____

ENROLLMENT NO: _____

ADMISSION CYCLE: _____

PROGRAMME CODE: MSCAST

COURSE CODE: MST-018

COURSE TITLE: MULTIVARIATE ANALYSIS

REGIONAL CENTRE CODE: _____

STUDY CENTRE CODE: _____

ADDRESS: _____

_____

_____

CONTACT NUMBER: _____

EMAIL ID: _____

DATE OF SUBMISSION: _____

School of Sciences

Indira Gandhi National Open University

Maidan Garhi, New Delhi-110068 (INDIA)

TUTOR MARKED ASSIGNMENT
MST-018: Multivariate Analysis

Course Code: MST-018
Assignment Code: MST-018/TMA/2025
Maximum Marks: 100

Note: All questions are compulsory. Answer in your own words.

1. State whether the following statements are true or false and also give the reason in support of your answer: **(5×2=10)**

(a) The covariance matrix of random vectors $\underset{\sim}{X}$ and $\underset{\sim}{Y}$ is symmetric.

(b) If $\underset{\sim}{X}$ is a p-variate normal random vector, then every linear combination $\underset{\sim}{c}'\underset{\sim}{X}$, where $\underset{\sim}{c}_{p\times 1}$ is a scalar vector, is also p-variate normal vector.

(c) The trace of matrix $\begin{pmatrix} 3 & -2 \\ -2 & 6 \end{pmatrix}$ is 9.

(d) If a matrix is positive definite then its inverse is also positive definite.

(e) If $\underset{\sim}{X} \sim N_2\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$ and $\underset{\sim}{Y} \sim N_2\left(\begin{pmatrix} -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$, then

$$\underset{\sim}{X} + \underset{\sim}{Y} \sim N_2\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right).$$

2 (a) Let $\underset{\sim}{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ has the following joint density function

$$f(x_1, x_2) = \begin{cases} 4x_1x_2 & ,0 < x_1 < 1,\ 0 < x_2 < 1, \\ 0 & ,\text{otherwise.} \end{cases}$$

Find the marginal distributions, mean vector and variance-covariance matrix. Also, comment on the independence of X_1 and X_2.

(b) Let $\underset{\sim}{X} = \begin{pmatrix} \underset{\sim}{X}^{(1)} \\ \underset{\sim}{X}^{(2)} \end{pmatrix} \sim N_4\left(\underset{\sim}{\mu}, \Sigma\right)$, where $\underset{\sim}{\mu} = \begin{pmatrix} -4 \\ 1 \\ 4 \\ 0 \end{pmatrix}$ and $\Sigma = \left(\begin{array}{cc|cc} 2 & 2 & 3 & 0 \\ 2 & 1 & 2 & 0 \\ \hline 3 & 2 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$. Find the

$E\left(\underset{\sim}{X}^{(2)} \middle| \underset{\sim}{X}^{(1)} = \underset{\sim}{x}^{(1)}\right)$ and $\text{Cov}\left(\underset{\sim}{X}^{(2)} \middle| \underset{\sim}{X}^{(1)} = \underset{\sim}{x}^{(1)}\right)$. **(10×2=20)**

3 (a) Let $\underset{\sim}{X}$ be a 3-dimensional random vector with dispersion matrix

$$\Sigma = \begin{pmatrix} 4 & -2 & 0 \\ -2 & 4 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

Determine the first principal component and the proportion of the total variability that it explains.

(b) Let $\underset{\sim}{X} \sim N_4(\mu, \Sigma)$, where $\underset{\sim}{\mu} = \begin{pmatrix} 3 \\ -2 \\ 1 \\ -2 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & -2 & 5 \end{pmatrix}$. Check the independence of the (i) X_2 and X_1 (ii) (X_2, X_4) and (X_1, X_3) (iii) (X_1, X_2) and (X_3, X_4).

(10×2=20)

4 (a) Consider the following data of 11 samples on 8 variables by Anscombe, Francis J. (1973):

x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4
10	10	10	8	8.04	9.14	7.46	6.58
8	8	8	8	6.95	8.14	6.77	5.76
13	13	13	8	7.58	8.74	12.74	7.71
9	9	9	8	8.81	8.77	7.11	8.84
11	11	11	8	8.33	9.26	7.81	8.47
14	14	14	8	9.96	8.10	8.84	7.04
6	6	6	8	7.24	6.13	6.08	5.25
4	4	4	19	4.26	3.10	5.39	12.50
12	12	12	8	10.84	9.13	8.15	5.56
7	7	7	8	4.82	7.26	6.42	7.91
5	5	5	8	5.68	4.74	5.73	6.89

If the vector $\underset{\sim}{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ and $\underset{\sim}{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}$, then obtain the sample covariance matrix between $\underset{\sim}{x}$ and $\underset{\sim}{y}$.

Source: Anscombe, Francis J. (1973). Graphs in statistical analysis. *The American Statistician*, **27**, 17–21. doi: 10.2307/2682899.

(b) Obtain the maximum likelihood estimator of the mean vector and variance-covariance matrix of the multivariate normal distribution.

(15+10=25)

5 (a) Define the following:

(i) Covariance Matrix

(ii) Mahalanobis D^2

(iii) Hotelling's T^2

(iv) Clustering

(v) Relationship between (ii) and (iii).

(b) If $\underset{\sim}{X} \sim N_3(\underset{\sim}{\mu}, \Sigma)$ with $\underset{\sim}{\mu} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 5 & 3 & 0 \\ 3 & 3 & -2 \\ 0 & -2 & 5 \end{pmatrix}$. Then find the joint distribution of $X_1 + 2X_2$, $2X_1 - X_2$ and X_3.

(15+10=25)